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ABSTRACT

Presented are student performance objectives, a student progress chart, and assignment sheets with objective and diagnostic measures for the stated performance objectives in College Algebra I. Topics covered include: sets; vocabulary; linear equations; inequalities; real numbers; operations; factoring; fractions; formulas; ratio, proportion, and variation; relations and Cartesian products; systems of equations; exponents; and quadratic equations. (MK)

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COLLEGE ALGEBRA I

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SE 029 347

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INTRODUCTION

During the Summer of 1974 criterion-referenced diagnostic tests were written for college arithmetic and college algebra. The materials included herewith are student performance objectives, student progress chart, assignment sheets/objective and diagnostic measures for each objective.

The materials were prepared for use at specific colleges in the 1974-75 school year and were so named to match the college numbering system.

College Arithmetic is the same as Math 60.

College Arithmetic and Pre-Algebra is the same as Math 50.

College Algebra I is the same as Math A.

College Algebra II is the same as Math D.

OBJECTIVES

MATHEMATICS A

TEACHER OBJECTIVES

1. SETS

1.1 SETS: NOTATION AND CONCEPTS. The student will identify the common symbols of set theory and will solve problems involving the use of the concepts: universal set, 1-1 correspondence, equality and equivalence of sets, and finite and infinite sets.

1.2 SETS: UNION AND INTERSECTION. Given two sets, the student will find the union and intersection of the sets.

1.3 SET BUILDER NOTATION. Given a set written in set builder notation, the student will write the same set in roster notation and, given a set written in roster notation, the student will write the same set in set builder notation.

1.4 THE SET OF REAL NUMBERS. Given a set of real numbers, the student will identify which members are irrational, rational, integral, and/or natural numbers, and will graph them on the real number line.

1.5 FINITE AND INFINITE. Given a set, the student will identify the set as finite or infinite.

2. VOCABULARY - TERMS

2.1 TRANSLATION OF PROBLEMS FROM WORDS TO SYMBOLS. Given verbal statements, the student will translate the statement into algebraic form.

5. LINEAR EQUATIONS

5.1 PROPERTIES OF EQUIVALENT EQUATIONS. Given the addition, subtraction, multiplication, and division properties of equivalent equations, the student will determine which of these properties are used in changing a given linear equation to a particular equivalent equation.

5.2 SOLUTION SETS. Given a linear equation in one variable, the student will find the solution set and graph the solution set on the number line.

5.3 GRAPHING IN ONE VARIABLE. Given one or more linear equation(s) the student will graph the solution set(s).

5.4 WORD PROBLEMS INVOLVING LINEAR EQUATIONS IN ONE VARIABLE. Given a word problem involving linear equations in one variable, the student will translate the problem into an algebraic equation, find the solution set of the equation, and choose the members that satisfy the conditions of the problem.

6. INEQUALITIES.

6. SOLUTION SETS FOR INEQUALITIES AND THEIR GRAPHS. Given an inequality in one variable, the student will find the solution set. Given the solution set of an inequality in one variable, the student will graph it on a real number line.

2.2 TRANSLATION OF PROBLEMS FROM SYMBOLS TO WORDS. Given problems in symbolic notation, the student will translate the problem into verbal form.

2.3 BASIC VOCABULARY. The student will define and identify prime numbers, exponents, algebraic terms, and numerical coefficients.

3. REAL NUMBERS.

3. FIELD POSTULATES. Given an example of the use of a field axiom, the student will name the field axiom that applies.

4. OPERATIONS WITH REAL AND LITERAL NUMBERS.

4.1 ADDITION AND SUBTRACTION OF ALGEBRAIC EXPRESSIONS. Given two or more polynomials, the student will add or subtract these polynomials by combining like terms.

4.2 MULTIPLICATION OF POLYNOMIALS. Given two polynomials, the student will divide by either a monomial or a binomial.

4.3 DIVISION OF REAL AND LITERAL NUMBERS. Given a polynomial, the student will divide by either a monomial or a binomial.

4.4 REMOVING GROUPING SYMBOLS. Given an algebraic expression with grouping symbols and simplify by collecting like terms.

7. FACTORING AND SPECIAL PRODUCTS.

7.1 MONOMIAL FACTORS. Given a polynomial of two or more terms, the student will identify and factor out the largest common monomial factor.

7.2 FACTORING THE DIFFERENCE OF TWO SQUARES. Given a binomial in the form of the difference of two squares or a binomial that can be changed into this form by factoring out a common monomial factor, the student will factor it completely.

7.3 TRINOMIALS. Given a trinomial with integral coefficients that is factorable, the student will factor completely.

7.4 FACTORING BY GROUPING. Given a polynomial that is factorable by grouping, the student will group terms properly and factor completely.

8. FRACTIONS.

8.1 REDUCTION. Given a reducible algebraic fraction, the student will reduce the fraction to a simpler equivalent form by removing one or more common factors from the numerator and denominator.

8.2 LEAST COMMON MULTIPLE. Given a set of polynomials, the student will find the least common multiple of that set.

8.3 LOWEST COMMON DENOMINATOR. Given a set of algebraic fractions, the student will find the lowest common denominator of that set.

8.4 EQUIVALENT FRACTIONS. Given an algebraic fraction, the student will change the fraction into an equivalent form by finding the

building factor used in the denominator and multiplying the numerator by that same factor.

8.5 ADDITION AND SUBTRACTION OF ALGEBRAIC FRACTIONS. Given two or more algebraic fractions, the student will find the sum and/or difference and simplify, if possible.

8.6 MULTIPLICATION. Given two or more algebraic fractions, the student will find the product and simplify, if possible.

8.7 DIVISION. Given two algebraic fractions, the student will find the quotient and simplify, if possible.

8.8 COMPLEX FRACTIONS. Given a complex fraction, the student will find its equivalent simple fraction reduced in the lowest form.

9. FRACTIONAL EQUATIONS

9. EQUATIONS CONTAINING FRACTIONS. Given a linear fractional equation or a fractional equation that simplifies to a linear equation, the student will find the solution set.

10. CHANGING THE SUBJECT OF A FORMULA

10. CHANGING THE SUBJECT OF A FORMULA. Given a formula, the student will solve for the indicated variable.

11. RATIO, PROPORTION, AND VARIATION.

11.1 RATIO, PROPORTION, AND APPLICATIONS. The student will determine the ratio of two given natural numbers; he will translate verbal statements to proportions and solve proportions.

11.2 VARIATIONS AND APPLICATIONS. Given two or more variables related in direct, inverse, or joint variation, the student will write the equation showing the appropriate relationship, and will solve problems applying variation.

12. RATIO, RELATIONS, AND CARTESIAN PRODUCTS

12.1 DEFINITIONS. The student will define the following concepts: relation, function, Cartesian product, domain, and range.

12.2 FUNCTIONAL NOTATION. The student will find the value of a function for a specified element of the domain.

12.3 GRAPHING LINEAR FUNCTIONS. Given a linear function and its domain, the student will graph the function in the Cartesian plane.

13. SYSTEMS OF EQUATIONS

13.1 SOLUTION OF A SYSTEM OF LINEAR EQUATIONS BY GRAPHING. Given a system of linear equations in two variables, the student will find the solution set by graphing.

13.2 SOLUTION OF A SYSTEM OF LINEAR EQUATIONS BY ADDITION. Given a system of linear equations in two variables, the student will find the solution set by the method of addition or subtraction.

13.3 SOLUTION OF A SYSTEM OF LINEAR EQUATIONS USING THE SUBSTITUTION METHOD. Given a system of linear equations in two variables, the student will find the solution set by the method of substitution.

13.4 APPLICATIONS OF SYSTEMS OF LINEAR EQUATIONS. Given a stated problem, the student will translate the problem into a system of linear equations in two variables and find the solution set of the problem.

13.5 SYSTEMS OF LINEAR INEQUALITIES. Given a system of linear inequalities in two variables, the student will graph its solution set.

14. EXPONENTS.

14.1 INTEGRAL EXPONENTS. Given algebraic expressions involving integral exponents, the student will completely simplify the expression so that the answer will contain only positive exponents.

14.2 SIMPLYING AND COMBINING RADICALS. Given radicals which are at most square or cube roots, the student will simplify and/or combine.

14.3 FRACTIONAL EXPONENTS AND THEIR RELATIONSHIP TO RADICALS. Given an algebraic expression involving fractional exponents, the student will write the expression using radical notation and simplify. Given an algebraic expression involving radicals, the student will write the expression in fractional notation.

14.4 EQUATIONS INVOLVING RADICALS. Given an equation involving square roots of linear algebraic expressions in one variable, the student will find and check the solution set.

14.5 EQUATIONS INVOLVING RADICALS. Given an equation involving square roots of linear algebraic expressions in one variable, the student will find and check the solution set.

15. QUADRATIC EQUATIONS

15.1 SOLUTION OF QUADRATIC EQUATIONS WITH NO LINEAR TERM. Given a quadratic equation in one variable in which there is no linear term, the student will find the solution set.

15.2 SOLUTION OF QUADRATIC EQUAITONS BY FACTORING. Given a factorable quadratic equation in one variable, the student will find the solution set by applying the theorem: if $ab = 0$ then $a = 0$ or $b = 0$.

15.3 SOLUTION OF QUADRATIC EQUATIONS BY COMPLETING THE SQUARE. Given a quadratic equation whose roots are real, the student will find the solution set by the method of completeing the square.

15.4 SOLUTION OF QUADRATIC EQUATIONS BY FORMULA. The student will memorize the quadratic formula and use it to find the solution sets of quadratic equations in one variable.

15.5 APPLICATIONS INVOLVING QUADRATIC EQUATIONS. Given a stated problem which involves a quadratic equation (including the Pythagorean Theorem), the student will correctly write the quadratic equation in one variable which interprets the problem, will find the solution set of the equation, and check for the elements in the solution set which correctly solve the problem.

STUDENT PROGRESS CHART

MATH A

STUDENT PROGRESS CHART

STUDENT'S NAME

sets								real-literals							factoring				fractions																functions				equations				exponents				quadratics			
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1.5 Finite - Infinite																			8.5 Addition - Subtraction																				13.5 Inequalities				14.5 Equations				16.5 Applications			
1.6 Field Postulates																			8.6 Multiplication																													
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2.2 Symbols to Words																			8.8 Complex																													
2.3 Vocabulary																																															
3.1 Solution Sets - Graphs																			9.1 Equations																													
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UNIT TESTS

TOPIC A-1.1

SETS: NOTATION AND CONCEPTS

OBJECTIVE:

● To use the given symbols, terms, and phrases correctly: $U, \cap, \in, \notin, \emptyset, \subseteq$, universal set, 1 to 1 correspondence, equality, and equivalence of sets.

A. Answer the following questions.

1. True or False:
   a. $3 \notin \{1,2,3,4,5\}$
   b. $\{2,5\} \subseteq \{1,2,3,4,5\}$
   c. $\{a,a,b,c\} = \{a,b,c\}$
2. Which symbol stands for the union of two sets?
3. Given $A = \{a,b\}$, find all subsets of A.
4. Given $B = \{1,2,3\}$, $C = \{a,b,c\}$, $D = \{2,3\}$.
   a. Can any two of the sets be put in a 1 to 1 correspondence? If so, which two?
   b. Are any two of the sets equivalent? If so, which ones?
5. A given set is called the __?__ set for all of its subsets.

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-1.2

SETS: UNION AND INTERSECTION

OBJECTIVE:

- To find the intersection and/or union of two sets.

  Example: $\{1,3,5,7\} \cap \{1,5,9,13\} = \{1,5\}$

A. Copy and perform the indicated operation.

1. $\{2,4,6,8,10\} \cap \{4,8,12,16\}$
2. $\{2,4,6,8,10\} \cup \{4,8,12,16\}$
3. $\{a,b,c\} \cap \{e,f,g,h\}$
4. $\{\text{all apples}\} \cup \emptyset$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-1.2

B. True or false:

1. The union of two sets is itself a set.
2. The intersection of two sets contains only those members which are common to both sets.
3. It is customary not to repeat a member of a set.
4. $\emptyset = \{0\}$

TOPIC A-1.3

SETS: SET BUILDER NOTATION

OBJECTIVES:

- Given a set written in set builder notation, to write the same in roster notation if possible.

  Example: $\{x|x \geq 6,\ x \text{ is a natural number}\} = \{6,7,8,...\}$

- Given a set in roster notation, to write the same set in set builder notation.

  Example: $\{1,2,3,4,5\} = \{x|x \leq 5,\ x \text{ is a natural number}\}$

A. Given the Universal set $\{0,1,2,3,4,5,6,7,8,9,10\}$.

1. Copy and change to roster notation.
   a. $A = \{x|x \text{ is an odd number}\}$
   b. $B = \{x|3 < x \leq 8\}$
2. Copy and change to set builder notation.
   a. $C = \{3,6,9\}$
   b. $D = \{5,6,7,8,9,10\}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-1.3

B. 1. Given A = {first 5 letters of the alphabet}. Write this in roster notation.

2. Express in words, the statement, $M = \{x|x > 3\}$.

3. What natural numbers are between 6 and 9?

4. Is $3 \in \{x|x > 3\}$?

TOPIC A-1.4

SETS: THE SET OF REAL NUMBERS

OBJECTIVES:

- To identify the different types of numbers that make up the set of real numbers.
- To classify the integers, rationals, and irrational numbers.

A.

1. Copy the following list of real numbers, circle the integers, underline the rationals and draw a box around the irrationals.

$\frac{3}{4}$	$-\frac{5}{1}$	$\sqrt{20}$	$-\sqrt{16}$	0
-4	18.6	π	$\frac{21}{5}$	$\frac{21}{3}$

2. By definition: A __?__ number is a number of the form $\frac{a}{b}$ where a and b are __?__ and b __?__ 0.

3. Draw the real number line and place the following numbers on it.

$\sqrt{3}$	$-\frac{1}{2}$	0	-0.6
$2\sqrt{3}$	3	$-\frac{18}{5}$	$-\sqrt{25}$

4. If a real number is not a terminating or repeating decimal, then it is called a(n) __?__ number.

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-1.4

B. 1. From the following list identify the numbers which are perfect squares.

3	7	9	12	16	25	30
36	45	49	60	64	75	81

2. Give another numeral for the following.

a. $\sqrt{1}$ b. $\sqrt{16}$ c. $\sqrt{49}$ d. $\sqrt{100}$

3. True or false.

a. $-6 > -3$ b. $-\frac{1}{16} > -\frac{1}{17}$

TOPIC A-1.5

FINITE AND INFINITE SETS

OBJECTIVE:

- To distinguish a finite set from an infinite set.

A. Designate each of the following sets as finite or infinite.

1. $\{x|3 < x < 24\}$, where x is an integer
2. $\{x|3 \leq x \leq 24\}$, where x is a real number
3. $\{1,2,3...\}$
4. $\{2,4,6,...,200,000\}$
5. $\{x|x$ is a grain of sand at Pismo Beach$\}$
6. $\{x|\frac{1}{1001} < x < \frac{1}{1000}\}$, where x is a rational number

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-1.5

B. Answer the following questions.

1. What is the largest number in $\{1,2,3,...\}$?
2. For the set $\{x|2 < x \leq 18\}$, where x is a real number:
   a. What is the smallest member
   b. What is the largest member?
3. True or false: $\{x|1 < x < 2\} = \emptyset$ if x is an integer.
4. True or false: $\frac{5}{6} > \frac{5}{7}$
5. True or false.
   a. All rational numbers are real.
   b. All real numbers are integers.
   c. All integers are rational.
   d. Between 6 and 10 there are 3 integers.

TOPIC A-2.1

TRANSLATION OF PROBLEMS FORM WORDS TO SYMBOLS

OBJECTIVE:

- To translate a given statement into an algebraic expression.

  Example: The product of two numbers represented by x and y: xy

A. Copy and write an algebraic expression for each of the following statements.

1. The sum of four times a and 5.
2. The difference when 3 is subtracted from two times the number represented by x.
3. One-fourth the product of 5x and y.
4. The quotient when x is cubed and then divided by 2.

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Translate into an algebraic expression.

1. The sum of a and 3.
2. The difference when 2 is subtracted from x squared.
3. One-half the product of 7 and m.
4. The result of dividing 4 times x by 3.

TOPIC A-2.2

TRANSLATION OF PROBLEMS FROM SYMBOLS TO WORDS

OBJECTIVE:

● To translate a given algebraic expression into a verbal statement.

A. Copy and write a verbal statement for each of the following algebraic expressions.

1. $x + 3x$
2. $2x^2 - y$
3. $3x\ (x - y)$
4. $\frac{x^3 + 2}{2}$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-2.2

B. Write a statement for the following.

1. $m + n$

2. $a + b - c$

3. $3xy^2$

4. $\frac{7y}{2}$

TOPIC A-2.3

BASIC VOCABULARY

OBJECTIVE:

- To define fundamental concepts.

A. Copy and define the following.

1. a prime number
2. a term
3. an exponent
4. a factor
5. a numerical coefficient

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-2.3

B. Fill in the blanks.

1. List the prime numbers between 4 and 20. ____?____
2. In the algebraic expression $4x + 3y + 2$; $4x$, $3y$, and 2 are called ____?____.
3. Given x^4. The numerical coefficient is ____?____.
4. Given $3 \cdot 5 = 15$; 3 and 5 are called ____?____.
5. Given $-5x^3$; the 3 is called the ____?____.

TOPIC A-3

FIELD POSTULATES

OBJECTIVES:

● Given examples of the field axioms, to state the name of the axiom that applies.

Examples:

(1) $5(a + b) = 5a + 5b$ — distributive axiom

(2) $x + (-x) = 0$ — additive inverse

A. Copy and state the name of the field axiom that applies.

1. $a + b = b + a$ — ?
2. $x \cdot 1 = x$ — ?
3. $3(x + y) = 3x + 3y$ — ?
4. $(3a)p = 3(ap)$ — ?
5. $(b + y) + 0 = b + y$ — ?
6. $(a + b) + c = c + (a + b)$ — ?
7. $-3x + 3x = 0$ — ?
8. $(3 + 2x) + y = 3 + (2x + y)$ — ?

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-3

B. Copy and answer as true or false.

1. $3 + 2 = 2 + 3$ is an example of the commutative axiom of addition.
2. $5 \cdot 7 = 7 \cdot 5$ is an example of the associative axiom of multiplication.
3. $y + 0 = y$ is an example of the additive inverse axiom.
4. $5 + (-5) = 0$ is an example of the additive inverse axiom.
5. $3(a + b) = 3a + 3b$ is an example of the distributive property.
6. $3a\ (\frac{1}{3a}) = 1$ is an example of the multiplicative identity axiom.

C. Copy and check (✓) the words that are used in the field axioms for real numbers.

1. _____ absolute value
2. _____ associative
3. _____ additive inverse
4. _____ coefficient
5. _____ exponent
6. _____ distributive
7. _____ prime
8. _____ identity for addition

TOPIC A-4.1

ADDITION AND SUBTRACTION OF ALGEBRAIC EXPRESSIONS

OBJECTIVE:

- To add and subtract similar terms and simplify.

  Example: $8x^2 + 2y^2 - x^2 - (-3y^2) = 7x^2 + 5y^2$

A. Copy and simplify the following.

1. $x + 4y + 5x - y$
2. $(4y^3) - (5x^2) + (-6y^3) + (3x^2)$
3. $x^2 - y + 2x^2 + 3y + 3x^2 - 2y$
4. $2a + 3b - 4c + 5a - 6b + 3c + (-3a) + b + c$
5. $(5rst) - (-7rst) + 6r$
6. Add: $5x - 6y + 3z$
   $2x - 8y + 7z$

7. Subtract: $5x - 6y + 3z$
   $2x - 8y + 7z$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-4.1

B. Copy and simplify.

1. $3x^2 + 4x^2$
2. $5y + (-7y)$
3. $3a^2 - (-8a^2)$
4. $(-3xy + (-8xy)$
5. Subtract: $\begin{array}{r} 11a \\ \underline{5a} \end{array}$
6. Subtract: $\begin{array}{r} -8x \\ \underline{-10x} \end{array}$
7. $2a + 3b + (-3a) - 3b$

C. Copy and answer either true or false.

1. $3b^2$ and $-7b^2$ are similar terms
2. $2x$, $2x^2$, and $2x^3$ are similar terms
3. $3 + 4y = 7y$
4. $2x^2 + 3x^2 = 5x^2$
5. $2a + a + 5a = 7a$

D. Copy and simplify.

1. $(+7) - (-3)$
2. $(-6) + (-5)$
3. $(-9) - (+2)$
4. $-3 - 4 + 5 - 6$
5. $8 - (-8)$

TOPIC A-4.2

MULTIPLICATION OF POLYNOMIALS

OBJECTIVE:

- To multiply monomial and polynomial expressions.

  Example: $(-4x^2y)(3xy^3) = -12x^3y^4$

A. Copy, multiply, and simplify the answer if possible.

1. $(3xy^4)(-5x^2y^2)$
2. $(rst)(-3rst)(r^2t^2)$
3. $(3x - 5y)(2x + 3y)$
4. $(3y^2 - 1)(3y^2 + 1)$
5. $3x^2(4x^2 - 5x + 1)$
6. $(3a + 5b)(2a - b)$
7. Multiply: $4a + 7b$ by $3a - b$
8. Multiply: $3x^2 - 4x + 7$ by $2x - 3$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-4.2

B. Copy and multiply.

1. $(x^2)(x^5)$

2. $(-2y)(-3y^2)$

3. $(-a^2)(a^3)(a)$

4. Multiply: $3x^2 - 2x + 1$ times $2x$

C. Copy and multiply.

1. $4(-3)$

2. $(-3)(-5)$

3. $(-1)(-2)(-3)$

4. $(-2)(+5)(-3)$

TOPIC A-4.3

DIVISION OF REAL AND LITERAL NUMBERS

OBJECTIVE:

- To divide polynomials by monomials and binomials.

Example: $\frac{12a^3 - 8a^2 + 4a}{4a} = 3a^2 - 2a + 1$

Example: $2x + 1\overline{)6x^2 - 5x - 4}$ $\quad 3x - 4$

A. Copy and divide as indicated.

1. $\frac{15x^4 + 6x^2 - 3x}{3x}$

2. $\frac{-9a^3b^2 + 6a^2b - 3}{-3ab}$

3. $(15x^2 + 7xy - 4y^2) \div (3x - y)$

4. $2x + 3\overline{)6x^2 + 7x + 2}$

5. $\frac{x^3 - 1}{x - 1}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-4.3

B. Copy and divide.

1. $\frac{21y^3}{-3}$

2. $\frac{-12x^6}{4x^2}$

3. $\frac{4a^2 - 8}{4}$

4. $\frac{2x + 1}{2}$

5. $\frac{8x^2 + 6x + 2}{2}$

6. $x + 1\overline{)x^2 + 5x + 4}$

C. Copy and divide.

1. $\frac{+28}{+4}$

2. $\frac{-15}{+3}$

3. $\frac{35}{-7}$

4. $\frac{-18}{-6}$

5. $\frac{25x^2}{-5}$

6. $\frac{x^3}{x^2}$

TOPIC A-4.4

REMOVING GROUPING SYMBOLS

OBJECTIVE:

- To remove grouping symbols and simplify.

  Example: $2x(x + 3) - 4(x^2 - 2x) = -2x^2 + 14x$

  Example: $-y - (2y - 5) = y - 5$

A. Copy and simplify.

1. $-2y - (7 - 3y)$
2. $4a - 2(a + 3b) - (3 + 2a)$
3. $2x^2 + 3x(2x + 1) - 2x(x - 5) + 3x$
4. $x^2 - 2(x^2 - y^2) + (y^2 - x^2)$
5. $2(3 - x) - x - (x + 2)$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-4.4

B. Copy and simplify.

1. $-2(a + 5)$
2. $3x(4 - x)$
3. $-(x^2 - 2x - 3)$
4. $-(x^2 - 1) - (x^2 + 3)$
5. $-(2x + 1) + 3(x + 1) - 2(2x - 5)$

C. Copy and check (✓) the correct response.

1. $-2 - (x + 4) =$

____ a. $-2 - x - 4$
____ b. $-2 - x + 4$
____ c. $2 + (x + 4)$

2. $-(7y - 3) + 4(1 - y) =$

____ a. $-7y - 3 + 4 - y$
____ b. $7y + 3 + 4 - y$
____ c. $-7y + 3 + 4 - 4y$

3. $-x(5x - 2) + 2x(x - 1) =$

____ a. $-6x + 2x + 3x - 2x$
____ b. $-5x^2 + 2x + 2x^2 - 2x$
____ c. $-3x^2 + 2x^2 - 2x$

4. $-2 \cdot 3y - 4(x - 3) =$

____ a. $-6y - 8(x - 3)$
____ b. $-2 \cdot 3y - 4x + 12$
____ c. $6y + 8(x + 3)$

TOPIC A-5.1

PROPERTIES OF EQUIVALENT EQUATIONS

OBJECTIVE:

- To identify the properties of equivalent equation.

List of Properties of Equivalent Equations

1. Addition property of equivalent equations
2. Subtraction property of equivalent equations
3. Multiplication property of equivalent equations
4. Division property of equivalent equations

A. State the property of equivalent equations that is applied.

Original Equation	Equivalent Equation	Property of Equivalent Equation Applied
1. $4x - 5 = 3$	$4x = 8$	?
2. $4x + 7 = 2x - 1$	$4x = 2x - 8$	?
3. $\frac{2}{3}x = 6$	$2x = 18$	?
4. $-5x = 15$	$x = -3$	?
5. $2x + 6 = 12 - x$	$6x + 18 = 36 - 3x$	?

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-5.1

B. Given the equation listed below in the first column state the operation you would perform to obtain the equation in the second column.

Example:

Original Equation	Equivalent Equation	Operation Performed
$x - 4 = 8$	$x = 12$	Add four to both members of the equation

State the operation used to obtain the equivalent equations from the original equation.

Original Equation	Equivalent Equation	Operation Performed
1. $x + 6 = 9$	$x = 3$	?
2. $2x = x + 6$	$x = 6$	?
3. $3x = 12$	$x = 4$	?
4. $\frac{4}{5}x = 8$	$x = 10$	?
5. $2x - 4 = -x + 6$	$x = 10$	?

TOPIC A-5.2

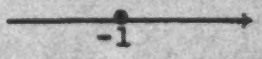
SOLUTION SETS

OBJECTIVE:

● To find and graph the solution set (roots) of linear equations in one variable.

Example: $4x + 2 = 3x + 1$

Solution: Solution set is {-1} or the root is -1.



A. Copy and graph the solution set (roots) of the following equations.

1. $x - 7 = 12$
2. $4x - 6 = 2x + 8$
3. $x = 4x - 30$
4. $2(x - 6) + 10 = 2(3x - 4) - 2(x - 3)$
5. $\frac{5}{6}x - 2 = 10$
6. $8 - \frac{3}{4}x = -12$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-5.2

B. Copy and find the solution sets (roots) of the following equations. Check your solution.

1. $4x - 2 = 10$
2. $3x - 5 = 7$
3. $2x + 5 = 3$
4. $8x = 5x - 9$
5. $4x = x - 6$

C. Copy and find the solution set (roots) of the following equations. Check your solution.

1. $4x = 20$
2. $\frac{1}{3} x = 2$
3. $-2x = 8$
4. $5x = -15$
5. $\frac{2}{5} x = 10$

D. Copy and find the solution set (roots) of the following equations. Check your solution.

1. $x + 8 = 10$
2. $x - 7 = 9$
3. $5x + 6 = 4x$
4. $4x - 3 = 3x + 2$

TOPIC A-5.3

GRAPHING LINEAR EQUATIONS IN ONE VARIABLE

OBJECTIVE:

● Given one or more linear equation(s) in one variable, to graph the solution set(s).

Example: $\{x|3x - 4 = 0\}$

A. Copy and graph the indicated set.

1. $\{x|5x - 3 = 12\}$
2. $\{x|x = 2 \text{ or } x = -8\}$
3. $\{x|2x = -x + 9\} \cap \{x|x = 8\}$
4. $\{x|5(x - 3) + 2(x + 4) = 0\} \cup \{x|\frac{1}{2}x = \frac{1}{3}\}$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-5.3

B. Copy and find the solution set.

1. $6x + 4 = 28$
2. $6(2x + 3) = 0$
3. $\frac{3}{5}x = 5$
4. $\frac{2}{3}x = -\frac{1}{6}$

C. Copy and answer the following questions.

1. $\{3\} \cup \{5\}$
2. $\{2,6\} \cap \{3,6\}$
3. $\{2,8\} \cap \{9,10\}$
4. $\{3,2,1\} \cap \emptyset$

TOPIC A-5.4

WORD PROBLEMS INVOLVING LINEAR EQUATIONS IN ONE VARIABLE

OBJECTIVES:

- To translate a word problem into an equation.
- To find the solution set of that equation.
- To choose those members of the solution set which satisfy the conditions of the stated problem.

Example: Sam invested $2000 in two separate accounts. One account pays 5% interest and the other pays 7% interest. At the end of one year $105 interest is earned. How much is invested in each account?

Solution: 1. $.05x + .07(2000 - x) = 105$

2. Solution set is {1500}

3. x = $1500 invested at 5%

   $2000 - x$ = $500 invested at 7%

TOPIC A-5.4

A. Solve the following problems by first writing an equation which fits the word problem.

1. One number is three more than another number. The sum of these two numbers is 19. What are the numbers?
2. Sue has six fewer dimes than quarters. The value of her money is $2.90. How many dimes does she have?
3. The sum of three consecutive odd integers is 63. What are these numbers?
4. One rectangle has a length which is five feet more than twice the width. A second rectangle is made by doubling the length of the first rectangle. The perimeter of the larger rectangle is 22 feet more than the smaller rectangle. Use the formula $P = 2\ell + 2w$.
5. Bridge mix and chocolate covered peanuts are used to make 16 pounds of mixed candy which sells for $.98 per pound. The bridge mix sells for $.90/lb. and the peanuts sell for $1.00/lb. How many pounds of each will be used?

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-5.4

B. Write an equation for each sentence.

1. The sum of x and y is 20.
2. The perimeter (P_1) is 3 feet more than the perimeter (P_2).
3. The differeice between 3x + 4 and 2x is 19.
4. The product of x and y is two less than their sum.
5. One-third the sum of x and twice y.

C. Do the following.

1. If n is the smallest of three consecutive odd integers, express all three integers in terms of n.
2. Give the penny value of k dimes.
3. Express the total penny value of x nickels and (x + 3) dimes.
4. Write an equation for the perimeter of a rectangle with side lengths of w and 3w - 2.

TOPIC A-6

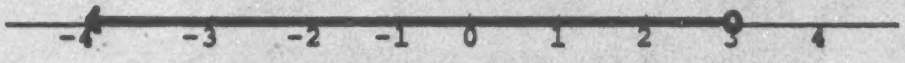
SOLTUION SETS FOR INEQUALITIES AND THEIR GRAPHS

OBJECTIVE:

● To find and graph the solution of linear enequalities (absolute value optional).

Example: Find and graph the solution set of $\{x|2x - 1 < 5\}$.

Solution: The solution set is $\{x|x < 3\}$.



A. Copy and find the solution set of the following and graph them on a number line.

1. $x - 3 < 2$
2. $3 - 2x < 1$
3. $\frac{x + 3}{5} < 2$
4. $\frac{1}{3} > 4 + 2x$
5. $|3x - 5| > 1$ (optional)
6. $|12 - x| < 3$ (optional)

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-6

B. Write out the theorems for operations on inequalities, ie. what happens when positive or negative numbers are added to both sides of an inequality? If both sides are multiplied or divided by a positive or negative number?

C. 1. Draw a real number line.

2. On a number line, graph the numbers less than 3.

3. Graph the numbers greater than -2.

4. Graph the numbers that are less than 5 and greater than -1.

5. One set of numbers is greater than -3. A second contains numbers less than 2. Graph the intersection of the two sets.

TOPIC A-7.1

MONOMIAL FACTORS

OBJECTIVES:

● To identify and factor the largest common mononial factor from a given polynomial.

Example: $12m^5 + 16m^4 - 20m^3 = 4m^3(3m^2 + 4m - 5)$

A. Copy and factor.

1. $20m^2n - 15mn^2$
2. $3x^5 - 9x^2 + 42x$
3. $4a^3b^3 + 8a^2b^2 - 18ab$
4. $2\pi r^2 + 6\pi rh$
5. $9a^2 + 18b^2$
6. $-2y^3 - 6y^2 - 2y$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-7.1

B. Copy and factor.

1. $7x + 14y$
2. $am - 2an$
3. $2a + 8b - 4ab$
4. $25x + 35x^2$
5. $-a - c - d$

C. Copy and fill in the parenthesis or blank.

1. $8x - 16y = 8 \cdot (\underline{\quad ? \quad})$
2. $3x^2 + 6x = \underline{\quad ? \quad}(a + 2)$
3. $5x^2y - 10xy^2 + 20y = 5y(\underline{\quad ? \quad})$
4. $-x^4 + x^3 - x^2 = -x^2 (\underline{\quad ? \quad})$
5. $a^3b^2 + 3a^3b^2 - 4ab = \underline{\quad ? \quad}(ab + 3a^2b - 4)$

TOPIC A-7.2

FACTORING THE DIFFERENCE OF TWO SQUARES

OBJECTIVE:

● To factor a polynomial which is the difference of two perfect squares.

Example: $x^2 - 4y^2 = (x + 2y)(x - 2y)$

A. Copy, factor completely, and check.

1. $4x^2 - 9$
2. $1 - 25x^2y^2$
3. $x^4 - 16$
4. $48y^2 - 75b^2$
5. $100a - ax^2y^2$
6. $\frac{a^2 - 9}{16}$
7. $(a + b)^2 - x^2$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-7.2

B. Copy and factor completely.

1. $x^2 - 9$
2. $y^2 - 25$
3. $4x^2 - 1$
4. $1 - b^2$
5. $2y^2 - 8$

C. 1. $x^2 - 16 = (x + 4)$ (____?____)

2. $y^2 - 49 =$ (____?____) $(y - 7)$

3. $2a^2 - 50 = 2(a + 5)$ (____?____)

4. $4x^2y^2 - 1 = (2xy - 1)$ (____?____)

5. $9 - 16y^2 =$ (____?____) $(3 + 4y)$

6. $1 - 4b^2 = (1 + 2b)$ (____?____)

TOPIC A-7.3

TRINOMIALS

OBJECTIVE:

● To factor into its prime factors a trinomial with integral coefficients.

Example: $x^2 + 7x + 12 = (x + 4)(x + 3)$

Example: $8x^2 - 2x - 15 = (4x + 5)(2x - 3)$

Example: $4x^2 - 20x + 16 = 4(x - 4)(x - 1)$

A. Factor the following trinomials into prime factors.

1. $x^2 - 22x - 23$
2. $1 - x - 2x^2$
3. $x^2 + 8xy + 15y^2$
4. $8t^2 - 10t + 3$
5. $2a^2 - 6a + 4$
6. $9x^2 - 30x + 25$
7. $3ax^2 - 6ax - 72a$
8. $3k^2 - 18k + 27$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-7.3

B. Factor the following trinomials into prime factors.

1. $x^2 - 5x - 6$
2. $m^2 + 6m + 9$
3. $x^2 - x - 20$
4. $x^2 - 13x + 42$
5. $10y^2 - 3y - 1$

C. Copy and complete the factoring of the following trinomials into prime factors.

1. $x^2 - 11x + 18 = (x - 2)(\quad ? \quad)$
2. $x^2 + 4x - 21 = (x - 3)(\quad ? \quad)$
3. $x^2 - 15x + 36 = (x - 3)(\quad ? \quad)$
4. $x^2 - 2x - 63 = (x + 7)(\quad ? \quad)$

TOPIC A-7.4

FACTORING BY GROUPING (OPTIONAL)

OBJECTIVE:

● Given a polynomial, to find one or more factors by appropriate grouping of the terms.

Example: $2ax - 2bx + 3ay - 3by =$

$2x(a - b) + 3y(a - b) =$

$(a - b)(2x + 3y)$

A. Copy and factor completely.

1. $bx + 2b + cx + 2c$
2. $x^2 - 5ax + xy - 5y$
3. $3ab + 2ax - 3by - 2xy$
4. $3x + 3y + (x + y)^2$
5. $(m - n)^2 - m + n$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-7.4

B. Copy, fill-in, and/or factor.

1. $(3x + 3y) + (4ax + 4ay) = 3(x + y) + 4a(x + y) = (x + y)(\quad ? \quad)$
2. $x(a - b) - y(a - b) = (\quad ? \quad)(\quad ? \quad)$
3. $3b(x - 4) - (x - 4)$
4. $(a + b)(a + b) + (a + b)$
5. $x^2y(a - 5) - 3(a - 5)$

C. Copy and fill-in.

1. $ax - bx = x(\quad ? \quad)$
2. $5x + 25xy = 5x(\quad ? \quad)$
3. $3bx - 3b = 3b(\quad ? \quad)$
4. $axy^2 - 7xy^2 = xy^2(\quad ? \quad)$
5. $-w - w^2 = -w(\quad ? \quad)$

TOPIC A-8.1

FRACTIONS: REDUCTION

OBJECTIVE:

- Reduce fractional expressions to lowest terms.

Example: $\frac{2x + 6}{4} = \frac{2(x + 3)}{2 \cdot 2} = \frac{x + 3}{2}$

A. Reduce the following fractions.

1. $\frac{4x^2 - 16x^3}{8x^2}$

2. $\frac{x^2 + 6x + 8}{x^2 - 4}$

3. $\frac{x^4 - 1}{(x + 1)^2}$

4. $\frac{x^2 - 1}{1 - 2x + x^2}$

5. $\frac{x^5 - x^3}{x - x^2}$

6. $\frac{4x^3y - 8x^2y^2}{4x^3y + 4x^2y^2}$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-8.1

B. Reduce the following fractions.

1. $\frac{2x + 10}{2}$

2. $\frac{7x - 21}{14}$

3. $\frac{x^2 + x}{x}$

4. $\frac{ab + a}{b + 1}$

C. Reduce the following fractions.

1. $\frac{8x^3}{4x^2}$

2. $\frac{3x^2}{9ax^3}$

3. $\frac{x^4y^3}{x^2y^2}$

4. $\frac{-6a^7b^4c^3}{12a^4bc^3}$

TOPIC A-8.2

FRACTIONS: LOWEST COMMON MULTIPLE

OBJECTIVE:

● Given a set of polynomials to find the lowest common multiple (LCM) of the members of the set.

Example: $\{9x^2, 18xy, 12y^2\}$ LCM is $36x^2y^2$

A. Find the LCM for the members of the following sets.

1. $\{10x^3y, 4xy, 3x^2y^2\}$
2. $\{5x + 15, x^2 - 25, 15\}$
3. $\{(x - 2)^2, 2x^2 - 8, 4x + 8\}$
4. $\{x^2 - 8x + 12, x^2 - 7x + 6, x^3 - 2x^2\}$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-8.2

B. Find the LCM for the following sets.

1. $\{x + 2, x - 2\}$
2. $\{3x + 12, 2x + 8\}$
3. $\{5x + 10, 7x + 14\}$
4. $\{x, x^3, x^2, x^5\}$
5. $\{xy^2, 12x^2y, 8xy\}$

C. Find the LCM for the following sets.

1. $\{3,4,6\}$
2. $\{9,6,12\}$
3. $\{40,20,15\}$
4. $\{8,20,50\}$

TOPIC A-8.3

FRACTIONS: LOWEST COMMON DENOMINATOR

OBJECTIVE:

● Given a set of fractions, to find the lowest common denominator (LCD) for the members of the set.

Example: $\{\frac{1}{x^2y}, \frac{5}{xy}, \frac{6}{xy^2}\}$ LCD = x^2y^2

A. Find the LCD for the following set of fractionsl expressions.

1. $\{\frac{3}{x^3y}, \frac{5}{2xy^2}\}$

2. $\{\frac{2}{x-1}, \frac{5}{x^2+x}\}$

3. $\{\frac{2}{x-4}, \frac{5}{x}, \frac{6}{x^2}\}$

4. $\{\frac{4x}{3(x-2)^2}, \frac{6-5x}{x^2-5x+6}\}$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-8.4

EQUIVALENT FRACTIONS

OBJECTIVE:

● To change a fraction to an equivalent fraction with a specified denominator.

Example: $\frac{4x}{9y} = \frac{?}{27x^2y}$

Solution: $\frac{4x}{9y} = \frac{12x^3}{27x^2y}$

A. Copy and find the missing numerator.

1. $\frac{2y}{3x^2} = \frac{?}{12x^4y^4}$

2. $\frac{x-2}{2} = \frac{?}{x(x-4)}$

3. $-2 = \frac{?}{6-x}$

4. $\frac{2x-1}{x-2} = \frac{?}{(x-2)(3x+5)}$

5. $\frac{x+y}{x-y} = \frac{?}{x(x-y)^2}$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-8.4

B. Copy and find the missing numerator.

1. $\frac{2}{3} = \frac{?}{12}$

2. $\frac{3}{5} = \frac{?}{10x}$

3. $\frac{4x}{7} = \frac{?}{21xy}$

4. $\frac{2}{3x} = \frac{?}{3x^2}$

C. Fill in the blanks.

1. 2xy must be multiplied by __?__ to obtain $12x^3y^4$

2. 6ac must be multiplied by __?__ to obtain $18a^4c^2$

3. a must be multiplied by __?__ to obtain $a^2 - a$

TOPIC A-8.5

ADDITION AND SUBTRACTION OF ALGEBRAIC FRACTIONS

OBJECTIVE:

- To find the sum and difference of algebraic fractions.

  Example: $\frac{2x}{3} + \frac{2x}{5} - \frac{16x}{15}$

A. Copy and find the sum or difference of the following algebraic fractions.

1. $\frac{5x + 1}{6} + \frac{3x}{8}$

2. $\frac{2}{x^2} + \frac{3}{x^3} - 4$

3. $\frac{4}{x - 4} - \frac{6 - x}{x^2 - 2x - 8}$

4. $\frac{2x}{x^2 - x - 2} + \frac{3x - 1}{x^2 + x}$

5. $\frac{3}{x^2 - 6x + 9} - \frac{1}{x^2 - 9}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-8.5

B. Copy and find the sum or difference of the following fractions.

1. $\frac{4}{3x} + \frac{2}{3x} + \frac{1}{x}$

2. $\frac{1}{x} - \frac{4}{x^2}$

3. $\frac{1}{a} + \frac{1}{b}$

4. $\frac{1}{x+2} - \frac{3}{x-1}$

C. Copy and find the sum or difference of the following fractions.

1. $\frac{2x}{3} + \frac{5x}{7}$

2. $\frac{x}{2} + \frac{3}{\quad}$

3. $\frac{6x}{5} + \frac{3x}{10}$

4. $\frac{2b}{5} - \frac{b}{3}$

D. Copy and find the sum of the following fractions.

1. $\frac{3x}{7} + \frac{2x}{7}$

2. $\frac{5x}{12} + \frac{5x}{12}$

3. $\frac{2xy}{5} + \frac{7xy}{5}$

TOPIC A-8.6

FRACTIONS: MULTIPLICATION

OBJECTIVE:

- To find the product of algebraic fractions.

Example: $\frac{2x}{3y} \cdot \frac{4x^2}{7y^2} = \frac{8x^3}{21y^2z}$

Example: $\frac{x}{x-1} \cdot \frac{x+1}{2x} = \frac{x+1}{2x-2}$.

A. Copy and find the following products and express in simplest form.

1. $\frac{-2x^2}{3yz} \cdot \frac{-9y^2}{8x} \cdot \frac{-z}{x^3}$

2. $\frac{3x-9}{x+1} \cdot \frac{x^2+x}{x^2-9}$

3. $\frac{x^3}{x^4-1} \cdot \frac{x+1}{x} \cdot \frac{x^2+1}{x^3}$

4. $\frac{2x^2-x-1}{3x^2+5x-2} \cdot \frac{x^2+5x+6}{x^2-1}$

5. $\frac{(x+2)^3}{6x^2-x-15} \cdot \frac{9x^2-30x+25}{x^2-4}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-8.6

B. Copy and find the following products. Express the result in simplest form.

1. $\frac{x + 2}{x} \cdot \frac{2x^2}{x + 2}$

2. $\frac{2x - 1}{xy} \cdot \frac{x^2y}{2}$

3. $\frac{2x - 3}{x + 1} \cdot \frac{x + 1}{2x + 3}$

4. $\frac{5x}{5x + 2} \cdot \frac{5x}{3}$

C. Copy and find the following products.

1. $\frac{5x}{2} \cdot \frac{3}{4y}$

2. $\frac{xy^2}{z^4} \cdot \frac{x^2y}{z^7}$

3. $\frac{6xy}{u^2v^2} \cdot \frac{2x^2}{9}$

TOPIC A-8.7

FRACTIONS: DIVISION

OBJECTIVE:

● To find the quotient of algebraic fractions and express in simplest form.

Example: $\frac{5x}{6y} \div \frac{15x^2}{2} = \frac{1}{9x}$

Example: $\frac{x^2 - x}{4} \div \frac{x - 1}{x} = \frac{x^2}{4}$

A. Copy and find the following quotients.

1. $\frac{x^2 + x}{x - 4} \div \frac{x^2 - 1}{x^2 - 8x + 16}$

2. $\frac{x^2 - x - 2}{x^2 - 4} \div \frac{(x - 2)^2}{(x + 2)^2}$

3. $\frac{x^2 - 8x + 15}{x - 1} \div \frac{x^2 - 6x + 9}{x^2 - 4x + 3}$

4. $\frac{x^2 - 2x - 35}{x^2 - 3x - 10} \div \frac{x^2 - 4x - 21}{x^3 + 2x^2}$

5. $\frac{x^3 - 1}{x + 1} \div \frac{x - 1}{x^3}$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-8.7

B. Copy and find the following quotients and express results in simplest form.

1. $\frac{(x - 2)(x + 2)}{x - 6} \div \frac{x - 4}{x}$

2. $\frac{x^2 - x}{2y^3} \div \frac{x}{y^4}$

3. $\frac{6x + 1}{x^2y^3} \div \frac{6x - 1}{xy}$

4. $\frac{x^3 - 2x^2}{8x} \div \frac{x^4}{2}$

C. Copy and find the following quotients and express results in simplest form.

1. $\frac{5x^2}{y} \div \frac{3x}{8}$

2. $\frac{2rs}{15t^2} \div \frac{8u}{3}$

3. $\frac{3a}{5b} \div \frac{2c}{7d}$

TOPIC A-8.8

FRACTIONS: COMPLEX

OBJECTIVE:

● To express a complex fraction as a simple fraction in lowest terms.

Example: $\dfrac{\frac{4a}{7b}}{\frac{3c}{8d}} = \dfrac{32ad}{21bc}$

Example: $\dfrac{\frac{1}{x} + \frac{1}{2x}}{\frac{1}{2x} - \frac{1}{x}} = -3$

A. Copy and simplify the following complex fractions. Express results in lowest terms.

1. $\dfrac{\frac{8x^3y}{ab^2}}{\frac{2xy}{ab}}$

2. $\dfrac{\frac{2x+1}{x}}{\frac{4x^2-1}{x^3}}$

3. $\dfrac{\frac{2x}{5} + \frac{x}{3}}{\frac{x}{2} - \frac{x}{3}}$

4. $\dfrac{\frac{2}{3x} + \frac{1}{4x}}{1 - \frac{1}{x}}$

5. $\dfrac{1 - \frac{1}{x^2}}{x - \frac{1}{x}}$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-8.8

B. Copy and simplify the following complex fractions. Express results in lowest terms.

1. $\dfrac{\frac{x^2}{3}}{\frac{x^3}{6}}$

2. $\dfrac{\frac{x^2 - 4}{2x}}{\frac{x + 2}{4x^2}}$

3. $\dfrac{\frac{3x + 1}{x^2}}{\frac{3x - 1}{x^4}}$

4. $\dfrac{\frac{x - 3}{x + 1}}{\frac{x^2 - 3x}{x + 1}}$

C. Copy and simplify the following fractions. Express results in lowest terms.

1. $\dfrac{\frac{2x}{3y}}{\frac{5r}{6s}}$

2. $\dfrac{\frac{-3}{4}}{\frac{x}{y}}$

3. $\dfrac{1}{\frac{x}{y}}$

TOPIC A-9

EQUATIONS CONTAINING FRACTIONS

OBJECTIVE:

● To find the solution set (roots of) equations in one variable involving fractions.

Example: $\frac{2}{3}x - \frac{(2x + 1)}{2} + 5 = 0$

Solution: $\left\{\frac{27}{2}\right\}$

A. Copy and find the solution set.

1. $\frac{1}{2}x = \frac{2}{3}x - 3$

2. $\frac{y + 8}{4} = \frac{5y - 2}{6}$

3. $\frac{2}{x} + \frac{4}{5} = 1\frac{1}{5}$

4. $\frac{2}{5}y - \frac{1}{3}y = 13 + \frac{1}{2}y$

5. $\frac{2(x - 1)}{3} = \frac{x + 1}{12} - \frac{1}{6}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-9

B. Copy and find the solution set.

1. $\frac{z}{4} + \frac{z}{2} = \frac{3}{4}$

2. $\frac{d}{6} + \frac{d}{3} = 28$

3. $\frac{1}{x} - \frac{2}{5} = 0$

4. $\frac{2(x + 3)}{3} = 4$

5. $10 - \frac{x}{2} = \frac{3}{4}x$

C. Copy and follow the directions.

1. Find the number by which you should multiply both members of the equation to clear fractions.

$$3 + \frac{x}{5} = \frac{2}{3}$$

2. Multiply both members by 12 and solve for x.

$$\frac{x}{3} - \frac{3x}{4} = \frac{5}{12}$$

3. Multiply both members by 6x and solve for x.

$$\frac{1}{2x} + \frac{2}{3} = 1$$

TOPIC A-10

CHANGING THE SUBJECT OF A FORMULA

OBJECTIVE:

- Given a formula, to change its subject.

  Example: $E = IR$ Solve for R.

  Solution: $R = \frac{E}{I}$

A. Copy and solve for the specified variable.

1.	$P = 2L + 2W$	Solve for L.
2.	$C = \frac{100W}{L}$	Solve for W.
3.	$F = \frac{aV}{V + b}$	Solve for b.
4.	$A = P + Prt$	Solve for P.
5.	$p = \frac{s}{s + f}$	Solve for s.

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-10

B. Copy and solve for x or y.

1. $5y = 15$
2. $y(a + 3) = 7$
3. $x(1 - b) = a$
4. $2x + ax = b$

C. Copy and write in an equivalent form without fractions.

1. $\frac{1}{a} = \frac{1}{x}$
2. $\frac{3}{4x^2} = \frac{1}{3x}$
3. $\frac{5}{b + x} = 3$
4. $\frac{4}{x - a} = \frac{3}{x - b}$

D. Copy and solve for x or y.

1. $x - 2.37 = 4.63$
2. $\frac{3}{4} y = 21$
3. $x + a = b$
4. $ay = c$

TOPIC A-11.1

RATIO, PROPORTION, AND APPLICATIONS

OBJECTIVES:

- To determine the ratio of two numbers.

  Example: The ratio of 3 to 5 is $\frac{3}{5}$.

- To solve problems involving proportion.

  Example: At the rate of 3 candy bars for 10¢, how many candy bars can you buy for 50¢?

  Solution: 15 candy bars

A. Copy and work the following.

1. Find the ratio of 20 to 8.
2. Find the ratio of 30 inches to 5 feet.
3. If 3 is to x as 12 is to 8, find x.
4. If 2 is to 5 as 6 is to (y - 1), find y.
5. On a certain map, 3" represents 50 miles. If two cities are 21" apart, find the number of miles between the cities.
6. A certain recipe requires 3 pints of milk for 8 servings. How many pints of milk are needed for 12 servings?

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-11.1

B. Copy and work the following.

1. Find the ratio of 2 to 5.
2. Given: $3:x = 5:8$, find x.
3. Given: $\frac{x+1}{6} = \frac{1}{2}$, find x.
4. If 12 eggs costs 72¢, use a proportion to find the cost of one egg.
5. Given the problem: At a certain time of day a 6 foot post casts an 8 foot shadow. What is the height of a pole which has a 15 foot shadow? Fill in the blanks:

$$\frac{x}{?} = \frac{?}{8}$$

TOPIC A-11.2

VARIATION AND APPLICATIONS

OBJECTIVES:

- To write equations of variation involving two or more variables.
- To apply variation in solving problems.

Example: If y varies directly as x and z, and y = 32 when x = 2 and z = 4, find y when x = 3 and z = 5.

Solution: $y = kxz$, then $32 = k(2)(4)$, so $k = 4$.

Now $y = 4xz$ so $y = 60$.

A. Copy and solve.

1. If y varies as x^2 and y = 36 when x = 3, what is the value of y when x = 5?
2. If y varies jointly as x and the square of z and y = 54 when x = 2 and z = 3, find y when $x = \frac{1}{3}$ and z = 4.
3. y varies jointly as x and the cube of z and inversely as a, y = x when x = 5, z = 2, and a = 4. Find y when a = 9, x = 10, and z = 3.
4. The volume of a gas varies directly as the temperature. If a gas has a volume of 450 ml. when the temperature is 300^0 C., what will be its volume when the temperature is 250^0 C.?

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-11.2

B. Copy and write the equation of variation.

1. y varies jointly as x and z.
2. F varies inversely with the square of d.
3. y varies jointly as x and the square of a and inversely as z.
4. V varies as the cube of r.

TOPIC A-12.1

FUNCTIONS, RELATIONS, CARTESIAN PRODUCTS: DEFINITIONS

OBJECTIVE:

● To define relation, function, Cartesian product, range, and domain.

A. Copy and define.

1. relation
2. function
3. the Cartesian product of sets A and B
4. range of the function
5. domain of the function

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-12.2

FUNCTIONAL NOTATION

OBJECTIVE:

- To use functional notation.

A. Copy and find the indicated:

1. If $f = \{(x,y) \mid y = f(x) = 2x - 3\}$, find:
   a. $f(5)$ b. $f(-2)$
2. If $h = \{(x,h(x)) \mid h(x) = 3x^2 + x\}$, find:
   a. $h(\frac{1}{2})$ b. $h(a)$
3. If $f = \{(1,2), (3,4), (5,6)\}$
   a. $f(3) =$ __?__ b. $f($ __?__ $) = 6$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-12.3

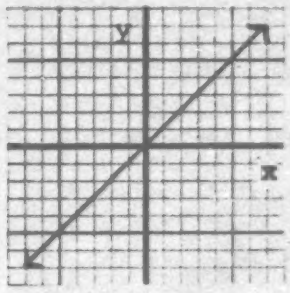
GRAPHING LINEAR FUNCTIONS

OBJECTIVE:

● Given a linear function and its domain, to graph the function in the coordinate plane.

Example: Graph $\{(x,y) | y = x, x \in R\}$

Solution:



A. Copy and graph the following when $x \in R$. R = {real numbers}.

1. $\{(x,y) | y = 2x - 3\}$
2. $\{(x,y) | 2x + 3y = 0\}$
3. $\{(x,y) | 3x - 2y = 6\}$
4. $\{(x,y) | y + 4 = 0\}$
5. $\{(x,y) | y = x + 1, x \geq 0\}$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-12.3

B. Copy and find for each equation two ordered pairs belonging to the solution set.

1. $y = x + 1$
2. $y = x - 2$
3. $y = 2x$
4. $y = \frac{3x + 1}{2}$

C. Copy and graph the following ordered pairs. (Use a separate coordinate system for each problem.)

1. (0,1); (3,1); (-4,1)
2. (2,3); (4,1); (-1,6)
3. (-2,-3); (0,1); (2,5)
4. (0,0); (-1,1); (2,-2)

D. Copy and solve for y.

1. $x + y = 5$
2. $x + 2y = 4$
3. $x - 3y = 7$
4. $4y - x = 2$

TOPIC A-12.4

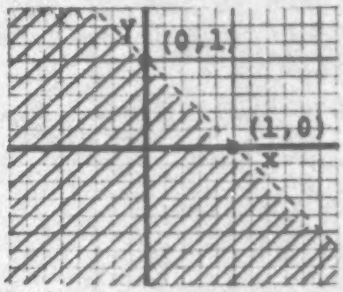
GRAPHING LINEAR INEQUALITIES

OBJECTIVE:

- To graph the solution set of a linear inequality.

  Example: $\{(x,y) \mid x + y < 1\}$

  Solution:



A. Copy and graph the following sets.

1. $\{(x,y) \mid y \leq x < -2\}$
2. $\{(x,y) \mid y > 1 - x\}$
3. $\{(x,y) \mid 2y + x \geq 2\}$
4. $\{(x,y) \mid 3y - 2x \leq 1\}$
5. $\{(x,y) \mid y - 3 < 0\}$
6. $\{(x,y) \mid 2x \geq y\}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-12.4

B. Is (0,0) an element of the following sets? Show your work.

1. $\{(x,y) \mid 3x - 6y \geq 10\}$
2. $\{(x,y) \mid x \geq 0\}$
3. $\{(x,y) \mid y < x - 8\}$
4. $\{(x,y) \mid 2x \leq 3y + 5\}$

C. Letting $x = 3$ and $y = 2$ fill in the blanks with $<$, $>$, or $=$.

1. $x - 2y$ ___?___ 0
2. $y + 2$ ___?___ $3x - 1$
3. $5x + 7$ ___?___ $2y + 18$
4. $5x + 7$ ___?___ $2y + 18$
5. $x + y + 3$ ___?___ $x - y - 1$

D. Copy and graph the linear equations on the x-y coordinate system.

1. $y = 2x + 1$
2. $x + y = -1$
3. $3x + 2y = 6$
4. $y - 3x = 0$
5. $\{(x,y) \mid y = 2\}$
6. $\{(x,y) \mid x + 1 = 0\}$

E. Copy and solve for y.

1. $y + x = 5$
2. $2y - 3x = 0$
3. $-3x + 2y = 7$
4. $5y - \frac{1}{2}x = 3$

TOPIC A-13.1

SOLUTION OF A SYSTEM OF LINEAR EQUATIONS BY GRAPHING

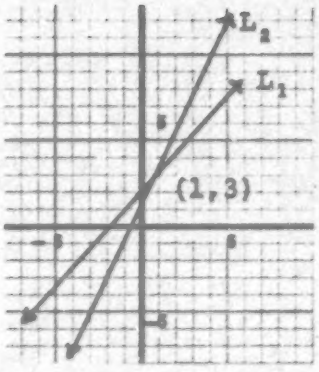
OBJECTIVE:

● To find the solution set of a system of equations using the method of graphing.

Example: $y = x + 2$

$y = 2x + 1$

Solution: (1,3)



A. Copy and find the solution set of the following systems of equations using the method of graphing. SHOW THE GRAPHS.

1. $y = x + 2$
   $y = 2x - 1$

2. $2x + y = 0$
   $y = 2$

3. $3x - y = 7$
   $2x - y = 5$

4. $3x + 2y = 12$
   $9x - 2y = 12$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-13.1

B. Graph the solution set of each of the following linear equations.

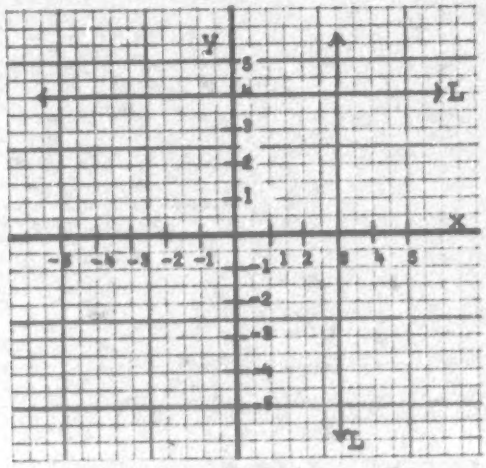
1. $y = 4x - 2$

2. $3x + 2y = 12$

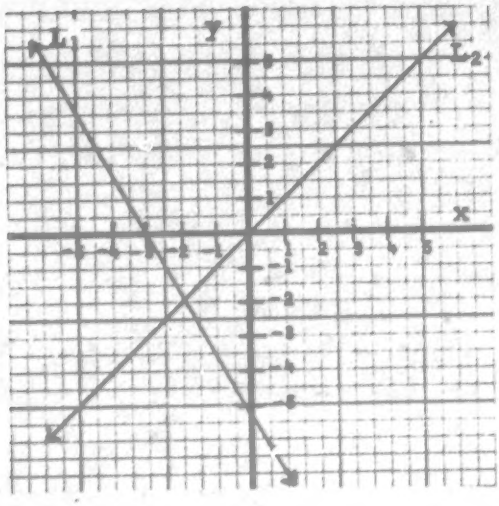
3. $3x + 5y = 15$

C. Given the graph of a system of linear equations find the solution set of that system.

1.



2.



TOPIC A-13.2

SOLUTION OF A SYSTEM OF LINEAR EQUATIONS BY ELIMINATION USING ADDITION AND SUBTRACTION.

OBJECTIVE:

To find the solution set of a system of equations by the method of elimination using addition or subtraction.

Example: $x + y = 3$

$x - y = 7$

Solution: The solution set is $\{(5,-2)\}$

Example: $2x - y = 0$

$x - 2y = -3$

Solution: The solution set is $\{(1,2)\}$.

A. Copy and find the solution set of the following systems of equations using the method of elimination by addition of subtraction.

1. $2x + 4y = 16$
   $2x - y = 1$

2. $3x + 4y = 12$
   $x - 2y = 4$

3. $3y + 2x = -5$
   $y = 6x - 5$

4. $\frac{1}{2}x = \frac{1}{3}y - 1$
   $\frac{1}{5}y = \frac{3}{4}x - 3$

5. $2x - 3y = 8$
   $3x - 5y = 26$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-13.2

B. Copy and find the solution set of the following systems of equations using the method of elimination by addition or subtraction. SHOW ALL WORK.

1. $x - y = 6$
   $x + y = 4$

2. $x - 2y = 8$
   $x + 2y = 2$

3. $2x - y = 2$
   $4x - y = 6$

4. $4x + 3y = 12$
   $2x + 3y = 12$

TOPIC A-13.3

SOLUTION OF A SYSTEM OF LINEAR EQUATIONS BY ELIMINATION USING SUBSTITUTION

OBJECTIVE:

● To find the solution set of a system of equations by the method of elimination using substitution.

Example: $2x + y = 6$

$y = 2x + 2$

Solution: The solution set is $\{(1,4)\}$

A. Copy and find the solution set of the following systems of equations using the method of elimination by substitution. SHOW ALL WORK.

1. $y = 2x$
   $5x - 2y = 3$

2. $4x = y + 2$
   $x - 2y = 4$

3. $2x + y = 8$
   $8x - 5y = 5$

4. $x + 2y - 4 = 0$
   $5x = 2 - 4y$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Copy and find the solution set of the following system of equations using the method of elimination by substitution.

1. $y = x$
   $x + y = 4$

2. $y = x - 1$
   $2x + y = 8$

3. $y = 2x + 3$
   $x + y = 0$

C. Copy and find the solution set of the following system of equations using the method of elimination by substitution.

1. $x = 3$
   $x + y = 2$

2. $y = 2$
   $x + 2y = 4$

TOPIC A-13.4

APPLICATIONS OF SYSTEMS OF LINEAR EQUATIONS

OBJECTIVE:

● To translate a stated problem into a system of two linear equations involving two variables and find the solution of the stated problem.

Example: The sum of two integers is 16. Their difference is 2. Find the integers.

Solution: The system of linear equations is:

$x + y = 16$ and $x - y = 2$

The integers are 9 and 7.

A. Solve the following problems using a system of two linear equations in two variables. SHOW ALL WORK.

1. The sum of two consecutive odd integers is 24. Find these integers.
2. The length of a rectangle is 4 times as large as its width. Its perimeter is 130 feet. Find the length and width of the rectangle.
3. John has 4 fewer nickels than quarters in his pocket. The total value of these nickels and quarters is $2.50. Find how many nickels and quarters John has.
4. Brad earned twice as much money as his brother last week. Together they earned $51. How much did each earn last week?

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Translate the given statements into two equations in two variables.

1. One number is three times as large as another number. The sum of these numbers is 36.
2. Kimber is 3 years older than Pam. The sum of their ages is 21.
3. Samuel has 2 more quarters than dimes. The total value of this coin collection is $4.00.
4. The width of a rectangle is 2 feet more than $\frac{1}{2}$ its length and its perimeter is 40 feet.

TOPIC A-13.5

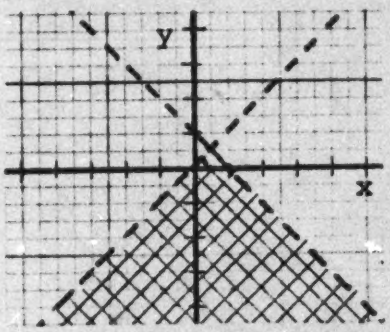
SYSTEMS OF LINEAR INEQUALITIES

OBJECTIVE:

● Given two linear inequalities involving intersections, to graph the solution set on the x-y coordinate system.

Example: $\{(x,y)|y < x\} \cap \{(x,y)|y < 1 < -x\}$

Solution:



A. Copy and graph the solution set of the following on the x-y coordinate system.

1. $\{(x,y)|y \leq 1 - x\} \cap \{(x,y)|y \geq x - 1\}$
2. $\{(x,y|2y - x < 2\} \cap \{(x,y)|x + y < 2\}$
3. $\{(x,y)|y \leq 2\} \cap \{(x,y)|x - y < 1\}$
4. $\{(x,y)|y + 1 > 0\} \cap \{(x,y)|x - 2 > 0\}$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-13.5

B. Copy and graph the solution set on the x-y coordinate system.

1. $\{(x,y) | y < x + 2\}$
2. $\{(x,y) | y \geq 2x - 1\}$
3. $\{(x,y) | y \leq 3 - 2x\}$

If unsuccessful, see Topic 12.4 section B.

C. Copy and graph the solution set on the x-y coordinate system.

1. $\{(x,y) | x < 1\}$
2. $\{(x,y) | y + 2 > 0\}$
3. $\{(x,y) | 2x + 5 \geq 8\}$
4. $\{(x,y) | 2y - 3 \leq 0\}$

D. Copy and graph the solution set on the real number line.

1. $\{x | x \leq 2\} \cap \{x | x \geq -1\}$
2. $\{x | x > 3\} \cap \{x | x < 0\}$
3. Find the set of all x greater than -2 and less than 5.
4. Find the set of all x less than 6 and greater than or equal to 10.

TOPIC A-14.1

INTEGRAL EXPONENTS

OBJECTIVE:

- To simplify algebraic expressions involving integral exponents.

Examples: 1. $\left(\frac{x^2y^6}{2x^3y^3}\right)^2 - \frac{y^6}{4x^6}$

2. $\frac{x^{-2}-3y^{-3}}{x^{-1}} - \frac{a(y^3-3x^2)}{x^2y^3}$

A. Copy and simplify (no negative exponents are to be in your answer).

1. $\left(\frac{x^3y^5}{xy}\right)$

2. $(2xy^{-2})^2$

3. $\left(\frac{-2x^3}{y^{-6}}\right)\left(\frac{x^0y^5}{8}\right)$

4. $\left(\frac{x^{-1}y^2}{x^3y^{-3}}\right)^{-2}$

5. $\left(\frac{x^{2n}}{(x^2)^{n+1}}\right)$

6. $(2^{-1} + 5^{-1})^{-1}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-14.1

B. Copy and simplify (no negative exponents are to be in your answer).

1. $(a^2b^3)^3$

2. $\frac{2y^{-6}}{y^{-2}}$

3. $\left(\frac{3^2x^4}{y^3}\right)\left(\frac{2y^6}{3x}\right)$

4. $2^{-1} + 4^{-2}$

5. $\left(\frac{3a}{4b}\right)^{-2}$

C. Simplify.

1. $\frac{x^6}{x^4}$

2. $6x^{-1}$

3. $(6x)^{-1}$

4. $(a^2)^4$

5. $(a^4)(a^3)$

6. $y^k \cdot y^2$

7. $(3x^2y^3)^3$

8. $(a + b)^2$

TOPIC A-14.2

SIMPLFYING AND COMBINING RADICALS

OBJECTIVES:

- To simplify and combine radicals.

Examples: 1. $\sqrt{50x^3y} = 5x\sqrt{2xy}$

2. $\sqrt{12a} + 3\sqrt{27a} - 2a\sqrt{3a} = 9a\sqrt{3a}$

3. $\sqrt[3]{-81} = -3\sqrt[3]{3}$

A. Copy and simplify.

1. $\sqrt{48x^2y}$
2. $\sqrt{125x^3y^5}$
3. $\sqrt[3]{16a^4b^6}$
4. $3\sqrt{24} + 2\sqrt{54} - \sqrt{96}$
5. $\sqrt{49x^2y^4} - 7xy^2$
6. $5\sqrt{3x} - \sqrt{27x} + 2\sqrt{75x}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-14.2

B. Copy and simplify.

1. $\sqrt{12}$
2. $\sqrt{18x}$
3. $\sqrt{a^7}$
4. $\sqrt[3]{24}$
5. $\sqrt{2} + 2\sqrt{50}$

C. Copy and simplify.

1. $\sqrt{36}$
2. $\sqrt{x^2y^5}$
3. $\sqrt[3]{27}$
4. $\sqrt{5} + 4\sqrt{5} - 2\sqrt{5}$
5. $\sqrt{49a^2b^6}$

TOPIC A-14.3

MULTIPLYING, DIVIDING, AND RATIONALIZING RADICALS

OBJECTIVE:

- To multiply, divide, and rationalize radicals.

Examples: 1. $\sqrt{2x} \cdot \sqrt{8x^3} = 4x^2$

2. $\dfrac{\sqrt{27a^4b^2}}{\sqrt{3ab^2}} = 3a\sqrt{a}$

3. $\dfrac{2}{3+\sqrt{3}} = \dfrac{3-\sqrt{3}}{3}$

A. Copy and simplify.

1. $\sqrt{5x^3} \cdot \sqrt{12x^2}$

2. $\sqrt{\dfrac{125}{y^3}}$

3. $\dfrac{\sqrt{64x^4y^5}}{\sqrt{128x^2y^2}}$

4. $\dfrac{4}{\sqrt{32x}}$

5. $(\sqrt{6} - \sqrt{2})(6 + \sqrt{2})$

6. $\dfrac{4}{\sqrt{6} - 2}$

7. $\dfrac{\sqrt{7} - 1}{\sqrt{7} + 1}$

8. $\sqrt[3]{2x^2} \cdot \sqrt[3]{4x^2}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-14.3

B. Copy and simplify.

1. $\sqrt{3x} \cdot \sqrt{12}$

2. $\dfrac{3}{\sqrt{x}}$

3. $\sqrt{\dfrac{2}{3x^2}}$

4. $2(\sqrt{3} + 1)$

5. $(\sqrt{3} + 2)(\sqrt{3} + 2)$

6. $\dfrac{2}{\sqrt{3} - 1} \cdot \dfrac{\sqrt{3} + 1}{\sqrt{3} + 1}$

C. Copy and simplify.

1. $\sqrt{5} \cdot \sqrt{5}$

2. $\dfrac{\sqrt{72}}{\sqrt{2}}$

3. $\dfrac{1}{\sqrt{5}}$

4. $\dfrac{\sqrt{24a^3}}{\sqrt{6a}}$

5. $\sqrt{2} \cdot \sqrt{3} \cdot \sqrt{6}$

6. $\sqrt{\dfrac{4}{9}}$

TOPIC A-14.4

FRACTIONAL EXPONENTS AND THEIR RELATIONSHIP TO RADICALS

OBJECTIVES:

- To write expressions in equivalent exponential and radical notation.

  Example: $x^{\frac{2}{3}}y^{\frac{1}{3}} = \sqrt[3]{x^2y}$

  $\sqrt{4x^3y} = 2x^{\frac{3}{2}}y^{\frac{1}{2}}$

- To simplify expressions involving fractional exponents.

  Example: $\left(\frac{x^{-\frac{3}{2}}}{y^{\frac{1}{2}}}\right)^2 = \frac{1}{x^3y}$

A. Copy and answer.

1. Write in radical form.

   a. $3^{\frac{2}{3}}$ b. $2^{-\frac{3}{4}}$

   c. $a^{\frac{1}{2}}b^{\frac{5}{2}}$ d. $(x - y)^{\frac{1}{3}}$

2. Write in exponential form.

   a. $\sqrt[3]{a^4b^2}$ b. $\sqrt[4]{(x + y)^3}$

   c. $\frac{1}{\sqrt{3ax}}$ d. $\sqrt{\frac{a^{-1}b}{c^2}}$

3. Simplify.

   a. $(-125)^{\frac{2}{3}}$ b. $81^{-\frac{3}{4}}$

   c. $\left(\frac{2^{\frac{1}{4}}x^{-\frac{1}{4}}}{2^{\frac{1}{2}}}\right)^{-4}$ d. $\frac{a^{-\frac{1}{4}}a^3}{a^{-\frac{1}{2}}}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-14.4

B. Copy and answer.

1. Write in radical form.

   a. $x^{\frac{1}{2}}$

   b. $(a + b)^{\frac{1}{3}}$

   c. $2^{-\frac{1}{2}}$

2. Write in exponential form.

   a. $\sqrt[3]{a^2}$

   b. $\sqrt{\frac{y}{4}}$

   c. $\frac{1}{\sqrt{x}}$

3. Simplify.

   a. $8^{\frac{2}{3}}$

   b. $\sqrt{4x^2y}$

   c. $\left(\frac{2^{\frac{1}{2}}}{2^{\frac{1}{4}}}\right)^4$

   d. $4^{\frac{1}{4}} \cdot 4^{-\frac{1}{4}}$

TOPIC A-14.5

EQUATIONS INVOLVING RADICALS

OBJECTIVE:

- To find the solution set of equations involving radicals.

  Example: $\sqrt{x + 1} = 4$ Find the solution set S.

  Solution: S = {15}

A. Copy and find the solution set.

1. $1 + \sqrt{x + 1} = 7$
2. $\sqrt{2x} - \sqrt{2x - 15} = 1$
3. $7 - \sqrt{2x} = 1$
4. $\sqrt{x + 6} = 1 - \sqrt{x - 3}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-14.5

B. Square the following.

1. $\sqrt{x}$
2. $\sqrt{3x}$
3. $\sqrt{x + 2}$
4. $\sqrt{x} - 2$
5. $1 + \sqrt{x - 6}$
6. $(a + 3)$

C. Find the solution set.

1. $\sqrt{x} = 3$
2. $1 + \sqrt{x} = 4$
3. $\sqrt{x} + 3 = 5$
4. $\sqrt{x + 1} = 5$

TOPIC A-15.1

SOLUTION OF QUADRATIC EQUATIONS WITH NO LINEAR TERM

OBJECTIVE:

- To solve a quadratic equation with no linear term.

  Example: $x^2 - 3 = 0$ Find the solution set, S.

  Solution: $S = \{-\sqrt{3}, \sqrt{3}\}$

A. Copy and find the solution set.

1. $x^2 = 25$
2. $a^2 - 8 = 0$
3. $5x^2 - 12 = 0$
4. $\frac{3}{7}x^2 - \frac{1}{5} = 0$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-15.1

B. True or false:

1. The quadratic equation, $x^2 - 3 = 0$ has 2 roots.
2. The graph of $\{x|x = \pm 3\}$ is two points because this set has only two members.
3. The solution set for $x^2 = 9$ is $\{3\}$.
4. $\{6,-6\} = \{\pm 6\}$

C. Copy and simplify.

1. $\sqrt{24}$
2. $\sqrt{\frac{8}{3}}$
3. $\pm 2\sqrt{40}$
4. $2 \pm \sqrt{16}$

TOPIC A-15.2

SOLUTION OF QUADRATIC EQUATIONS BY FACTORING

OBJECTIVE:

- To solve a quadratic equation by factoring.

  Example: $x^2 - x - 12 = 0$ Find the solution set S.

  Solution. $S = \{-3, 4\}$

A. Copy and find the solution set.

1. $(x - 6)(x + 7) = 0$
2. $6x(x - 3) = 0$
3. $6x^2 + 7x - 5 = 0$
4. $8x^2 = 10x$
5. $x(x - 3) = 4$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-15.2

B. 1. If $ab = 0$ then what conclusion can be drawn?

2. If $8x = 0$, what is the solution set?

3. True or false: If $x^2 - 3x = 0$ then the solution set is $\{\pm\sqrt{3x}\}$.

4. True or false: For the equation $5x^2 = x$, the solution set is $\{\frac{1}{5}\}$.

C. Factor completely.

1. $2x^2 - 4x$

2. $x^2 - 8x + 7$

3. $15x^2 + 11x - 12$

4. $12x^2 + 8x - 4$

TOPIC A-15.3

SOLUTION OF QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

OBJECTIVE:

- To solve a quadratic equation by completing the square.

  Example: $x^2 + x - 1 = 0$ Find the solution set, S.

  Solution: $S = \left\{\frac{-1 \pm \sqrt{5}}{2}\right\}$

A. Copy and find the solution set by completing the square.
SHOW ALL WORK.

1. $x^2 + 6x - 7 = 0$
2. $x^2 - 5x + 4 = 0$
3. $3x^2 + 6x - 5 = 0$
4. $\frac{1}{3}x^2 - 3x = \frac{1}{2}$

➧ HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-15.3

B. Choose the correct constant to make a perfect square trinomial in the left member (side). Add this constant to each side of the equation.

1. $x^2 - 10x = 6$
2. $x^2 + x = 3$
3. $2x^2 + 10x = -1$
4. $3x^2 - 11x = 2$

C. Copy and find the solution set.

1. $(x + 5)^2 = 16$
2. $(x - 3)^2 = 20$
3. $(x - \frac{3}{5})^2 = \frac{17}{25}$
4. $(x + \frac{5}{3})^2 = \frac{4}{9}$

D. Copy and find the solution set.

1. $x + 3 = \pm 8$
2. $x - 2 = \pm 8$
3. $x - \frac{5}{6} = \frac{\pm\sqrt{24}}{3}$
4. $x + \frac{7}{6} = \pm\frac{2}{5}$

TOPIC A-15.4

SOLUTION OF QUADRATIC EQUATIONS BY FORMULA

OBJECTIVE:

- To solve a quadratic equation using the quadratic formula.

  Example: $3x^2 + 7x - 5 = 0$ Find the solution set, S.

  Solution: $S = \left\{\frac{-7 \pm \sqrt{105}}{6}\right\}$

A. Copy and find the solution set by using the quadratic formula.

1. $x^2 + 5x + 3 = 0$
2. $6x + 7 = x^2$
3. $3x^2 = 8x + 2$
4. $\frac{x^2}{5} + \frac{x}{2} = 1$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC A-15.4

B. 1. Given $ax^2 + bx + c = 0$, which of the following is the quadratic formula?

a. $x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$ b. $x = \frac{b \pm \sqrt{b - 4ac}}{2a}$

c. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ d. $x = \frac{b \pm \sqrt{b^2 + 4ac}}{2a}$

2. For the following, identify a, b, and c as used in the quadratic formula.

a. $3x^2 = -8x + 2$

b. $x^2 + 2x - 3 = 0$

c. $8x^2 = 3x$

3. Simplify and state as two numbers.

a. $\frac{-2 \pm \sqrt{49}}{9}$ b. $\frac{4 \pm \sqrt{20}}{6}$

c. $\frac{-3 \pm \sqrt{16 - 4(0)}}{2}$ d. $\frac{5 \pm \sqrt{1}}{2}$

C. Evaluate the first two formulas in part B(1) if a = 3, b = -2, and c = -8.

TOPIC A-15.5

APPLICATIONS INVOLVING QUADRATIC EQUATIONS

OBJECTIVE:

- To solve problems involving quadratic equation.

  Example: The square of a certain positive number is 28 more than three times the number. Find the number.

  Solution: $x^2 = 28 + 3x$ The number is 7.

A.

1. A rectangle is two feet longer than it is wide. The area is 80 square feet. Find the dimensions.
2. Find two consecutive negative integers such that the sum of their squares is 41.
3. A certain number is increased by its reciprocal. The sum is $\frac{13}{6}$. Find the number and its reciprocal.
4. The hypotenuse of a right triangle is 97 feet. One leg is five feet more than the other. What is the length of the longer leg?

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!